Use of Statistically Designed Experiments in Wind-Tunnel Test Programs

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Very often in engineering problems, the number of variables is so large that the test cost and time available do not allow for the investigation of all the possible solutions. A properly designed statistical experiment can effectively examine all combinations of the variables while greatly reducing the number of tests required. This paper presents two typical statistically designed experiments and a discussion of the data analysis techniques. The examples are: 1) a parametric test of reverser geometry (three variables of three levels), and 2) an investigation of the important design parameters for several exhaust-nozzle schemes (16 variables of two levels).

Preliminary

BEFORE we go into the discussion of statistical test techniques, let us first define some of the language used.

1) A *variable* or *factor* is a geometric, aerodynamic, thermal, etc. quantity, whose change in magnitude is to be evaluated by the test; for example, the effect a change in exhaust-nozzle jet area or a change in external Mach number has on nozzle performance.

2) A *level* is the magnitude of each variable (e.g., a two-level change might be the variation in jet area from 5 to 6 ft^2, a three-level change might be tests conducted at M_N = 0.5, 0.7, and 0.9). Note that a three-level change can produce a curve whereas a two-level test can only produce a straight-line variation in the measured performance.

3) The term *confounded* is used to mean the indecisive results that confuse the indicated performance change with more than one variable. Fig. 1, for example, shows that a performance improvement from 0.92 to 0.97 is possible with a jet area change from 5.0 to 6.0 ft^2. The same matrix also shows that the same performance increase is possible for a Mach number decrease from $M_N = 0.7$ to $M_N = 0.5$. The confounded results do not show which variable is clearly responsible.

4) *Latin square* is the term used to describe the type of experiment which contains an equal number of variables and levels. A 3-factor latin square, for example, might contain the variables Mach number M_N, jet area (A_J), and exhaust-nozzle pressure ratio (NPR), each varied at three levels (Table 1 below) for a total possible number of combinations of $(3)^3$ or 27.

5) *Replicate* or *replication* describes the proportion of the total experiment which will be used. Many times, even statistical experiments must be abbreviated to minimize cost. A reduction in the full design experiment is known as a fractional replication. For better accuracy, experiments may be multiplied as well as fractionated.

6) *Multiple regression* involves a curve-fitting procedure in which coefficients are calculated for each variable according to an assumed equation form (i.e., quadratic, cubic, etc.). An equation that best fits the test data is derived, and can be used to predict trends in performance.

Table 1 Example of latin square

M_n	0.5	0.7	0.9
A_j	4 ft^2	5 ft^2	6 ft^2
NPR	2.0	3.0	4.0

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Introduction

As the requirements for improved aerodynamic performance increase and more sophisticated and complex solutions are evolved to meet them, the experimental means for evaluating these solutions becomes detailed and expensive. In many cases, the complexity of the device is such that analytical techniques for predicting results are inadequate, and the only suitable means of evaluating its performance is experimentally. More often than not, however, the possible combinations of the variables that should be investigated are so numerous that the time and costs involved in testing become prohibitive. It is here that both engineering judgment and statistically designed experiments play an important role. No statistical experiment is complete without proper engineering judgment. The engineer is forced to organize his test and to categorize the geometric and aerodynamic variables. He must decide how big a variation is required to show a significant change in performance. Too small a change will not give clear trends. Many times, the circumstances of the test makes it easier to make aerodynamic variations than to make geometric changes, thus influencing the layout of the test program. Statistical experiments usually require a broad range of variations. This is both good and bad. Configurations must be tested which often can be intuitively judged to be poor; however, to extend data to a higher level, a lower-level point must be established. These configurations can be deceiving if they are chosen based on intuition alone. Experience has shown that by covering this broad range of variations, new horizons can be uncovered.

The broad range of variations in a statistical test provides valid data for many configurations that might be useful in parametric studies when structural or other compromise solutions are necessary, whereas the "normal" engineering test usually requires a new test to evaluate the compromise. This parametric-type output can also be useful on other

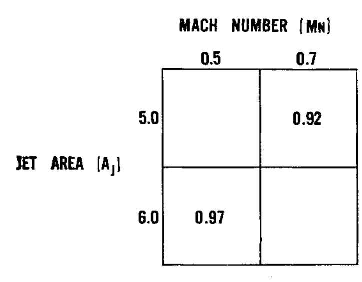

Fig. 1 Two-factor latin square. Numbers in boxes are the measured performance numbers, i.e., η, C_{VP}, drag, etc.

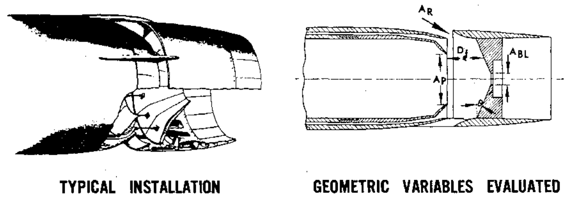

Fig. 2 Blow-in-door ejector and thrust reverser.

similar projects. In general, the normal engineering tests are so specific that they become limited in scope to one project. The statistical program tends to broaden the scope of the data. In very complex problems, where the analytical techniques are inadequate for accurate performance evaluation, the optimum configuration can be predicted experimentally. This then places a goal for the analytical backup, and limits its scope to "why is it optimum" rather than "what is optimum," thus making the task easier. The time saved may be applied to product improvement.

It must be clearly understood, of course, that this type of experimentation requires the keen judgment of a knowledgeable engineer. Not all programs can be conducted statistically. The expected output must be considered. When the data are expected to peak out over the range of variables, consideration should be given to three-level vs two-level variations. The following examples will describe how the statistical testing technique was used to resolve two problems of different complexity.

Example 1

To begin with, let us look at a relatively simple experiment to learn the mechanics of the technique. It was desired to evaluate the suppression-free thrust-reversing potential of a thrust reverser packaged in a typical Pratt & Whitney Aircraft blow-in-door ejector. As shown in Fig. 2, engineering background indicated that the variables for this exhaust-nozzle reverser scheme should be the bleed area produced by the reverser flap closure (A_{BL}), the reverse flow area (A_R) obtained by axially translating the shroud, and the flap-deflection angle (θ) which sets flap length. Additional variables that could have been considered, such as flap-hinge location, primary nozzle size (A_P), external Mach number, variations in external lines, etc., were maintained constant, to prevent the program from becoming too complex. The variables selected were considered to be the most influential and were expected to give the most significant trends. To allow the results to be more generally applicable, the area variations were ratioed to primary nozzle area A_P, thus making all variations essentially parametric. The test results, reverse thrust coefficient (C_{VR}) and flow coefficient (Q), are shown tabulated in Fig. 3 on the test matrix. Only nine of the 27 possible variable combinations were required. Why were only $\frac{1}{3}$ of the possible combinations tested? Looking at the heavily lined portion of the matrix in Fig. 3, note that it includes all the data with the variable θ at the 0° level. Note also that there is one test for each level of the other two variables A_R/A_P and A_{BL}/A_P, as shown in Table 2 below.

NOZZLE PRESSURE RATIO ~ P_{TP}/P_{AMB} = 2.0
FREE STREAM MACH NUMBER ~ M_∞ = 0

FLAP DEFLECTION ANGLE (DEGREES) ~ θ	0			10			22		
BLEED TO PRIMARY AREA RATIO ~ A_{BL}/A_P \ REVERSER TO PRIMARY AREA RATIO ~ A_R/A_P	1.88	1.46	0.88	1.88	1.46	0.88	1.88	1.46	0.88
0		(2) Q=0.948 C_{VR}=0.850				(6) Q=0.780 C_{VR}=0.856	(7) Q=0.955 C_{VR}=0.854		
0.047			(3) Q=0.816 C_{VR}=0.750	(4) Q=0.950 C_{VR}=0.750				(8) Q=0.950 C_{VR}=0.755	
0.157	(1) Q=0.948 C_{VR}=0.577				(5) Q=0.944 C_{VR}=0.570				(9) Q=0.890 C_{VR}=0.560

Q ~ NOZZLE FLOW COEFFICIENT
C_{VR} ~ REVERSE THRUST COEFFICIENT

Fig. 3 Three-factor latin square matrix for blow-in-door ejector-reverser test.

Table 2 Data tabulation for flap angle, θ = 0°

Box no.	A_R/A_P	A_{BL}/A_P	C_{VR}	Q
1	1.88	0.157	0.577	0.948
2	1.46	0	0.850	0.948
3	0.88	0.047	0.750	0.816
	Avg. C_{VR} = 0.726; Avg. Q = 0.940			

The average reverse thrust coefficient for θ = 0° is 0.726. The point to note here is that in this process, the effects of the high and low levels of both variables are averaged, and the median level is assumed. Following the same procedure for the boxes with θ = 10° and 22°, as indicated in Table 3 below, the effect of flap angle on reverse thrust coefficient and nozzle flow coefficient can be plotted as shown in Fig. 4.

Referring to this figure, the reverse thrust coefficient is essentially independent of flap angle. The nozzle flow coefficient, however, shows substantial nozzle suppression for the small flap angles and indicates that flap angles greater than 22° are required for obtaining desirable flow characteristics. Going back to Fig. 3, let's examine the row labeled A_{BL}/A_P = 0. Note that it too has tests that include one test at each level of the other two variables, as tabulated in Table 4 below.

The same is true for the rows labeled A_{BL}/A_P = 0.047 and 0.157. Using the same averaging procedure as before, for the θ = constant boxes, the variations of C_{VR} and Q with A_{BL}/A_P can be compiled, and are shown plotted in Fig. 5. Here a very significant change is noted for both variables. The slope of the reverse thrust coefficient curve [i.e., $C_{VR}/(A_{BL}/A_P)$] is -2 and indicates that not only is reverse thrust lost by the loss of airflow through the hole, but a forward thrust is gained by discharging it aft. The result is a loss in reverse thrust equivalent to twice the reversed airflow loss. The process is now repeated for the third variable; namely, the performance and flow coefficients are averaged for constant levels of A_R/A_P. The results, tabulated below in Table 5, and shown graphically in Fig. 6, indicate essentially no change in reverse thrust coefficient with reverse flow area.

Severe nozzle suppression, however, is shown for A_R/A_P less than 1.46. There is every indication that the point of suppression is probably sudden and that more points should be taken between 0.88 and 1.46. Additional points were eventually tested on another configuration in a later test, and the inflection point in the curve was determined to be at an area ratio A_R/A_P = 1.1.

This test is a good example of how the wrong selection of a variable can lead to confounding. As mentioned previously, the hinge point of the flap was held constant relative to the shroud, but the shroud was moved relative to the primary nozzle to vary the reverse flow area (A_R). This caused the distance from the primary nozzle to the flap to vary two ways: 1) with shroud translation and 2) with flap-angle variation. Thus, because the distance (D_f) from the primary nozzle to the flap, as shown in Fig. 2, was not selected as a variable,

Table 3 Data tabulation for flap angles, θ = 10° and 22°

Box no.	C_{VR}	Q	Box no.	C_{VR}	Q
4	0.750	0.950	7	0.854	0.955
5	0.570	0.944	8	0.755	0.950
6	0.856	0.780	9	0.560	0.890
	Avg. C_{VR} = 0.725; Avg. Q = 0.891			Avg. C_{VR} = 0.723; Avg. Q = 0.932	

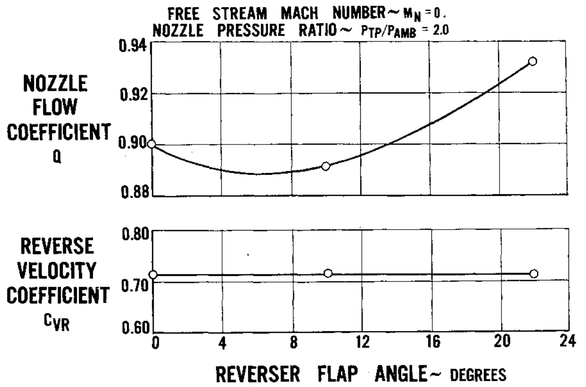

Fig. 4 Effect of reverser flap angle on reverse thrust performance in blow-in-door ejector-reverser test.

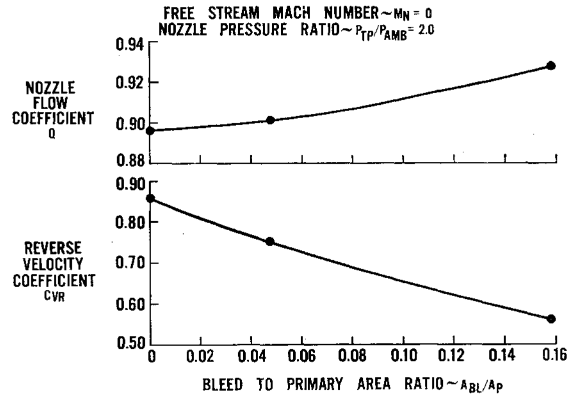

Fig. 5 Effect of bleed area on reverse thrust performance in blow-in-door ejector-reverser test.

the trends in performance and flow coefficient resulting from variations in D_f are confounded, and the true effects of changes in this parameter cannot be evaluated. In subsequent tests, the variable θ was replaced by variations in D_f.

Example 2

Introduction

In order to more fully understand the problems encountered in the design of a blow-in-door ejector nozzle, a simple analysis of its function is necessary. The basic principle is relatively simple but the flow problems encountered make the theoretical analysis difficult. A convergent-divergent (*C-D*) nozzle operating at low pressure ratios becomes greatly overexpanded, causing the exit pressure to drop below the ambient pressure. This produces a large base drag. It is possible to supply additional air to the nozzle at these low operating pressure ratios and prevent the overexpansion from occurring. When this is done, the *C-D* nozzle becomes an ejector. If the additional air is obtained from the freestream through doors in the cowling, the nozzle is called a blow-in-door ejector. (See Fig. 7.)

The calculation of nozzle performance involves a pressure balance between the two streams as well as an approximation of their mixing losses. The amount of air bled in through the blow-in doors is dependent upon the internal pressures; but the internal pressures are dependent on the amount and conditions of the air bled in. The resulting balance of conditions is a reasonably complex inlet-nozzle matching problem requiring an iterative routine for solution. Although the basic design parameters were pretty well established, correlation between theory and test had not been successful. Extensive experimental work of the "trial and error" nature had been carried out with some measure of success, and to back up the continuing theoretical efforts, a decision was made to try speeding up the experimental program through the use of statistical analysis.

The design variables were selected as coordinates describing the contours of the nozzle. The nozzle performance comparisons would be made at the same conditions as in previous trial-and-error-type tests; namely, freestream Mach number of 1.2 with nozzle pressure ratio of 4.0 and 2% secondary cooling flow. All the models had a maximum outside diameter of 4 in.

Table 4 Data tabulation for bleed area ratio = 0

Box no.	θ	A_R/A_P	C_{VR}	θ
2	0	1.46	0.850	0.948
6	10	0.88	0.856	0.780
7	22	1.88	0.854	0.955
	Avg. C_{VR} = 0.853; Avg. Q = 0.894			

Description of Models

Recognizing that the variables were to be geometric shape descriptions, the points shown in Fig. 8 were chosen. There were 16 in all. The longitudinal distances were measured from the end of the forebody and the radial distance from the centerline. To cut down the number of variables, the positions of points 3 and 2 were set at 20 and 60% of the distance from 4 to 1, and similarly points 6 and 8 were 60% of the distance from 5 to 7. The axial location of point 9 was left a variable. The aft section (shroud) was positioned relative to

Table 5 Data tabulation for effect of reverser area

Averages of box nos.	A_R/A_P	Avg. C_{VR}	Avg. Q
1, 4, 7	1.88	0.727	0.951
2, 5, 8	1.46	0.725	0.947
3, 6, 9	0.88	0.722	0.829

the forebody by point 5 as was the primary nozzle by point 10. It was decided to simplify the problem somewhat, and point 8 was eliminated making the line from 7 to 9 straight. With reference to Fig. 8, the final outcome was 13 variables as follows: L_1(R_1 fixed), R_2, R_3, R_4, (L_4 = 0), L_5, R_5, R_6, L_7, H_7, R_7, L_9, R_9, L_{10}, R_{10}. The type of statistical test chosen was a two-level (high and low) experiment comprised of 16 configurations. Its purpose was to screen the main effects and eliminate the variables with small contributions to performance. Before models could be built, numerical values had to be assigned to each variable and the high and low range selected. After much juggling to avoid impossible combinations, the variables listed in Table 6 were assigned taking care that the primary nozzle and afterbody structure were included in the envelope.

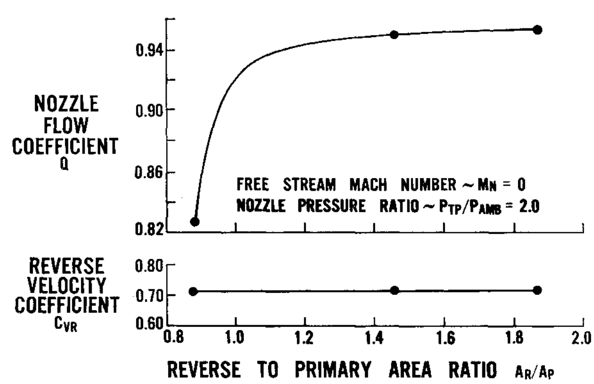

Fig. 6 Effect of reverse flow area on reverse thrust performance in blow-in-door ejector-reverser test.

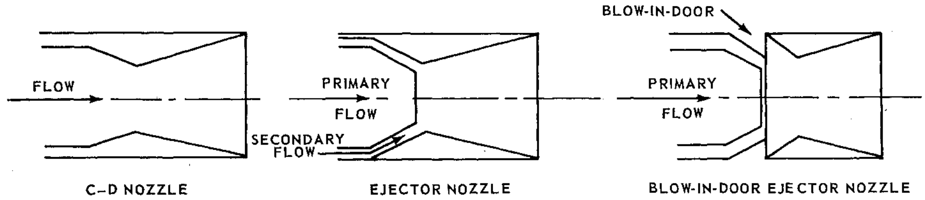

Fig. 7 Nozzle schematic drawing.

Results of Test I

Table 7 shows the design experiment and the nozzle performance coefficients (C_{FP}) recorded. The test data indicated that no variable could be eliminated because its contribution to performance was definitely small. In fact, all were confounded (interacting) with each other in such a manner as to make it impossible to tell which were of most importance. It is interesting to note that in this first test one model performed as well as the best trial and error model previously tested. The question remaining to be answered was: what is the best we can do? Not enough data were available to determine which combination of variables could give best performance or whether or not we even had the correct range of variables.

Table 6 Sizing of chosen variables in blow-in-door ejector test

Model symbol	Size, in. Hi	Size, in. Lo
L_1	6.6	4.5
R	1.95	1.85
R_3	1.85	1.75
R_4	1.75	1.40
L_5	0	−0.50
L_7	4.8	2.5
L_9	1.2	0.75
L_{10}	1.0	−0.50
R_5	2.0	1.85
R_6	1.85	1.75
R_7	1.70	1.55
R_9	1.70	1.55
R_{10}	1.2	0.98

Test II

The program had to be concentrated; some of the variables had to be eliminated and the scope of the program reduced in size. R_2, R_3, and R_5 were fixed at the high value because they tended to favor this side but did not show much difference either way. The effect of point 6 was ignored as all indications were that it had very little effect. Later this was better substantiated. Point 7 was fixed with L_7 high and R_7 low only to eliminate the effect of its variation and to avoid having to make new parts, and R_9 was allowed to vary out of the required pattern in hopes it would not affect the results. The remaining variables were as follows: L_1, R_4, L_5, L_9, L_{10}, R_{10}. The experiment selected was designed to evaluate each variable and all possible second-order interactions as well as some third-order interactions. Thirty-two tests were required. Figure 9 shows the matrix with all 48 test points spotted on it. The suspected high-performance regions have been marked off (i.e., high or low variable size),

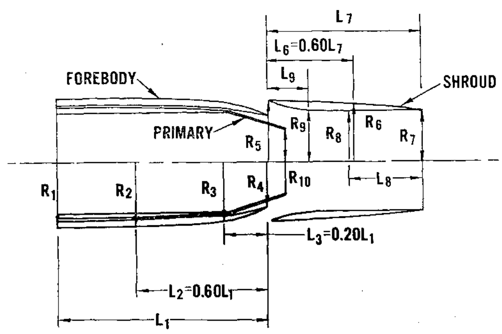

Fig. 8 Assignment of variables in blow-in-door ejector-nozzle test. L_5 ~ distance between trailing edge (TE) of forebody and leading edge (LE) of shroud (positive when shroud is downstream). L_{10} ~ distance between TE of forebody and TE of primary (positive when primary is upstream). Forebody designated by number; shroud designated by letter.

and some extra tests are shown which were used to confirm the conclusions. Reasonably good agreement was obtained, but when a multiple regression analysis was attempted using all the data, a poor correlation fit was obtained and large errors were also found when old model data were used to check the results. The obvious conclusion was to go back and pick up some of the variables that had been dropped out. Another 32-test experiment was designed for seven other variables.

Test III

In this test the nozzle primary size (R_{10}) was fixed at the high value. This would be the case normally, as the engine

Table 7 Design experiment I of blow-in-door ejector test (all interactions confounded)

Test	Model	L_1	L_5	L_7	L_9	L_{10}	R_2	R_3	R_4	R_5	R_6	R_7	R_9	R_{10}	C_{FP}
1	1A	H	H	H	H	H	H	H	H	H	H	H	H	H	0.915
2	2B	L	L	H	H	H	H	L	L	L	H	H	H	L	0.700
3	3C	H	L	H	L	H	L	H	H	L	L	H	L	L	0.663
4	4D	H	H	L	L	L	H	L	L	L	L	H	H	H	0.920
5	5E	H	H	H	H	L	H	H	L	L	L	L	L	L	0.820
6	6F	L	L	L	L	L	H	H	H	H	L	H	H	L	0.615
7	7G	L	L	L	L	H	H	H	L	L	H	L	L	H	0.853
8	8H	L	H	H	L	H	L	L	L	H	L	H	L	H	0.825
9	9I	L	H	L	H	H	L	H	L	H	L	L	H	L	0.734
10	10J	H	L	H	L	L	L	H	L	H	H	L	H	H	0.902
11	11K	H	H	L	L	H	H	L	H	H	H	L	L	L	0.705
12	12L	L	L	H	H	L	H	L	H	H	L	L	L	H	0.892
13	13M	L	H	H	L	L	L	L	H	L	H	L	H	L	0.690
14	14N	H	L	L	H	H	L	L	H	L	L	L	H	H	0.851
15	15O	L	H	L	H	L	L	H	H	L	H	H	L	H	0.857
16	16P	H	L	L	H	L	L	L	L	H	H	H	L	L	0.793

L_1 (H,L)	L_5 (H)	L_9 (H)	L_{10} (L)	R_4 (H,L): H	H	H	H	L	L	L	L
				R_9 (H): H	H	L	L	H	H	L	L
				R_{10} (H): H	L	H	L	H	L	H	L
H	H	H	H	0.915			0.880			0.885	
H	H	H	L			0.914		0.917			0.820 0.838
H	H	L	H	0.895			0.705		0.799		
H	H	L	L		0.776			0.920 0.891			
H	L	H	H	0.851		0.880					0.812
H	L	H	L				0.714	0.913		0.905	0.793
H	L	L	H		0.551		0.663	0.878			
H	L	L	L	0.817				0.898 0.902	0.794		
L	H	H	H			0.887			0.734		0.792
L	H	H	L			0.857	0.811			0.890	
L	H	L	H		0.736			0.865			
L	H	L	L	0.889	0.690				0.793		
L	L	H	H				0.723		0.700	0.893	
L	L	H	L			0.892 0.898					0.813
L	L	L	H	0.814					0.779	0.853 0.825	
L	L	L	L		0.615 0.613			0.900			

Fig. 9 Test II matrix for blow-in-door ejector test. (Symbols in parentheses denote favored levels.)

requirements would dictate the primary area. L_1 was fixed high as all indications were that this was best. R_2 and R_3 were kept fixed at the high value as in test II and R_6 was also ignored as before. L_7 was allowed to vary the same as R_5 so that both effects were marked. It was assumed that R_5 had a much larger effect than L_7 so that the latter could be ignored. The variables used were: R_4, L_5, R_5, R_7, R_9, L_9, L_{10}, and the test was designed to evaluate more of the two- and three-order interactions. Figure 10 shows the 7-factor matrix with all of the data spotted on it. The best combinations of variables were becoming more and more evident, especially when sketches of each model were examined in the order of performance level. Some additional check runs were made to confirm the results, with good agreement. The regression analysis was conducted using all the data points that had been taken for all three tests and, although still not accurate, gave better results. However, because of all the interactions no attempt was made to determine the best model. The results indicated that several were high-performing and the best was probably not unique. One indication was that blow-in-door area was a better variable than some of the multiorder interactions that had been used, so the areas were calculated for all of the tests and another regression analysis made. The results were similar but pointed out that this area was definitely an important parameter.

Figure 11 shows this latest regression equation and the comparison between test and calculated performance. It was felt that the basic task that had been originally outlined was

R_4 (H,L)	R_5 (L)	R_7 (H-H) (L-L)	R_9	L_5 (H): H	H	H	H	L	L	L	L
				L_9 (H): H	H	L	L	H	H	L	L
				L_{10} (L): H	L	H	L	H	L	H	L
H	H	H	H	0.915 0.913					0.847		
H	H	H	L				0.884			0.793	
H	H	L	H			0.895	0.889 0.895			0.814 0.808	0.817
H	H	L	L	0.887 0.904	0.914			0.880	0.892 0.898 0.890		
H	L	H	H				0.861			0.825	
H	L	H	L	0.852	0.857				0.820		
H	L	L	H	0.851				0.851	0.884		
H	L	L	L				0.858			0.820	
L	H	H	H		0.917	0.865			0.913		0.880
L	H	H	L		0.875	0.825		0.861			
L	H	L	H		0.858	0.865	0.891	0.857		0.878	0.902 0.900
L	H	L	L	0.885	0.890	0.840		0.893	0.905		0.871
L	L	H	H		0.879		0.920	0.880			0.898
L	L	H	L			0.875					0.890
L	L	L	H			0.873					0.877
L	L	L	L		0.908			0.903		0.853	

Fig. 10 Test III matrix for blow-in-door ejector test. (Symbols in parentheses indicate desired levels.)

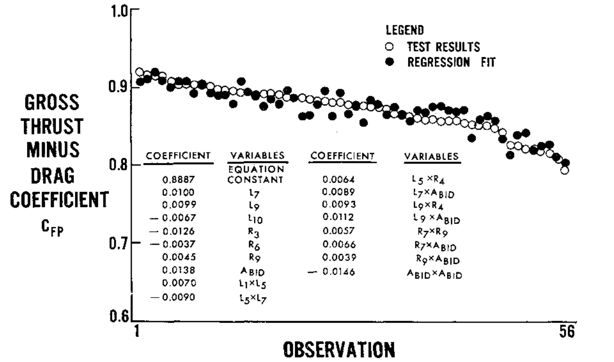

Fig. 11 Results of final multiple regression analysis of blow-in-door ejector test.

accomplished. The feasibility of using statistics with engineering judgment to reach a conclusion more rapidly than by trial and error had been demonstrated and the regression equation gave a ready means for checking the performance of various blow-in-door ejector (BIDE) designs. The basic task had been completed but now the big job was ahead: developing the nozzle at several operating conditions and Mach numbers. The regression analysis indicated that A_{BID}, R_3, and L_9 were the most influential variables and that R_6 had the least effect. This points to the importance of the inlet flow conditions and indicates that the external shroud drag will almost take care of itself. In a development program many of the other variables become fixed for structural or mechanical considerations. Once this is done an even smaller, more concentrated effort can be placed on the remaining variables.

Discussion

One advantage of statistical testing over conventional cut-and-try methods is the wealth of data taken in the poor regions as well as the better ones. This provides tradeoff factors when structural or other compromises are necessary. It also makes optimization at several Mach numbers or flight conditions possible by offering a wide range of variables at each condition so that the best over all may be selected. It is also possible to obtain optimum data long before theory or trial and error can determine it, and it is known that the configuration is optimum therefore providing a definite end. The theoretical efforts can then be expended in determining why it is optimum, thus placing a goal to these studies as well. No statistical program would be complete without the engineer to guide it and determine required variables. The proper use of statistics as a tool in the hands of an engineer can be most rewarding.

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